A New Diagrammatic Approach to Quantum Information

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1 Abstract

We introduce a new diagrammatic approach to quantum information, that we call *holographic software* [1]. Our new approach is a small modification of previous diagrammatic approaches, yet our small change leads to a great deal of new understanding and to a large number of new insights and motivation. Its application leads us to new protocols, designed in a "topological" way.

(1) The fundamental idea to use a diagrammatic notation to describe tensor manipulation originated in the work of Penrose [2]. This work has been extensively developed by Abramsky, Coecke, and their coworkers, yielding many diagrammatic representations of tensors and other algebraic structures in tensor networks [3, 4].

(2) Many diagrams have topological meaning, and its importance in quantum information was recognized in the pioneering work of Kitaev, Freedman, Larsen, Wang, Kauffman, and Lomonaco [5, 6, 7, 8, 9]. In addition, the topological model of Kitaev, Levin, and Wen provide powerful tools in quantum information [5, 10].

Manin and Feynman introduced the concept of quantum simulation for quantum systems [11, 12, 13]. In our framework we simulate quantum networks using a topological model PAPPA constructed in [14]. This fits into the philosophy of (2). Our holographic software allows us to translate between the topological approach and the algebraic approach. Thus we obtain new diagrammatic interpretations of algebraic structures. This fits into the philosophy of (1).

Communication: In communication we aim to change the location of quantum information, without changing the data. We simulate this process in a purely topological manner, that we call *topological simulation*.

When we simulate the bipartite teleportation, we recover in a natural way the resource state, measurement, and Pauli matrices [14, 1]—as well as the protocol of Bennett and coworkers [15]. When we simulate more complicated processes—by following topological intuition—then we find that our framework also leads to something new.

What is new in our model? Our topological model consists of charged strings as the basic elements. These charges represent the type of particles. For examples, in the qubit case, the charge 0 or 1 indicate that the particle is boson or fermion respectively. In the qudit case, the charge has d possible values representing parafermions.

In previous work, diagrams satisfy topological isotopy. In our work, diagrams represent fermions or parafermions and they satisfy a general type of topological isotopy, that we call *para isotopy*, see [14, 1]. In this way, we obtain a natural diagrammatic representation and topological interpretation of Pauli matrices, of measurements, and of Jordan-Wigner transformations.

Kauffman and Lomonaco suggested that the braid produced entanglement [8]. In our approach, we introduce the new notion in quantum information: the string Fourier transform $\mathfrak{F}_s$ (SFT). The SFT becomes our powerful tool, missed in previous approaches, that leads to maximal entanglement of states. (Note SFT is different from the standard quantum Fourier transform F. We define SFT geometrically; it generalizes F.)

Not only is our approach with SFT new, but it appears more robust. We show that the action of SFT on the multipartite ground state gives a the state $|\mathrm{Max}\rangle$,

$$\mathfrak{F}_s|\vec{0}\rangle = |\mathrm{Max}\rangle. \tag{1}$$

This is related to $|\mathrm{GHZ}\rangle$ [16] by the local transformation given by the quantum Fourier transform F:

$$|\mathrm{Max}\rangle = (F \otimes \cdots \otimes F)|\mathrm{GHZ}\rangle. \tag{2}$$

We give the diagrammatic representation for the n-qudit $|\mathrm{Max}\rangle$ in Figure 1. In the case of 2-qubits, $|\mathrm{Max}\rangle$ simplifies to become the usual resource state, the Bell state.

We use $|\mathrm{Max}\rangle$ as a multipartite resource state in protocols. The topological feature of $|\mathrm{Max}\rangle$ leads to new designs of protocols for multipartite communication. For example in [1] we give a generalization of the BVK protocol [17].

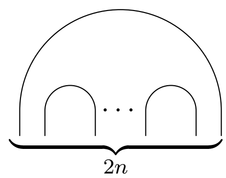

Figure 1: Diagrammatic representation of the n-qudit $|\mathrm{Max}\rangle$. There are $2n$ boundary points at the bottom.

We show that many interesting transformations, such as controlled transformations, have a property that we can say that they are *topologically compressed*. This topological feature is compatible with that of $|\mathrm{Max}\rangle$. From this observation, we design an optimized protocol to teleport such transformations for multiple persons in multiple parties. We call this protocol multipartite compressed teleportation (MCT) [18].

Summary: Each paragraph in the "what's new" section represents one new result. We give many others in our papers. We continue to find new applications to quantum information, and we believe that our model will play an important role in the future of the field.

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