# Multipartite Compressed Teleportation

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In a previous paper we introduced holographic software for quantum networks, inspired by work on planar para algebras. This software suggests the definition of a compressed transformation. Here we utilize the software to find a CT protocol to teleport compressed transformations for multipartite quantum communication.

## I. INTRODUCTION

In a previous paper we introduced holographic software for quantum networks [1], inspired by work on planar para algebras [2]. We follow that notation: $X, Y, Z$ denote qudit Pauli matrices, $F$ is the Fourier transform, and $G$ is the Gaussian. Recently the authors of [3] cite teleportation as the "most promising mechanism for a future quantum internet." Here we utilize the holographic software to find a *compressed teleportation* (CT) protocol for multipartite quantum communication.

In our software we represent a 1-qudit transformation as a "two-string" diagram, namely a diagram that has two input points and two output points. Many important transformations only act as "one-string" transformations, such as Pauli matrices, or the controlled transformation on the control qudit. We call such transformations *compressed.*

An original teleportation protocol was given by Bennett et al [4]. An optimized teleportation protocol was recently given in [5], and in [6, 7] one finds extensive references. Experimental work on long-distance teleportation has been achieved [8, 9]. All of these protocols are designed for two persons.

In this paper we give the new lossless CT protocol to teleport compressed transformations for multipartite quantum communication. This generalizes many teleportation protocols, and demonstrates a new paradigm for teleportation networks that fundamentally differs from the cooperative networks for two persons teleportation [10, 11].

Comparing CT with the teleportation in [4], our protocol reduces the cost by 50%, since it uses $2n$ noiseless channels to construct $n$ resource states for two persons, but only $n$ noiseless channels to construct one resource state for $n$ persons. Furthermore, the *Quantum Science Satellite* being built by Pan and his coworkers will test teleportation at record-breaking distances [12, 13]. This provides an opportunity to test CT in space, and establish a benchmark for future multipartite protocols in the developing global quantum network.

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## II. MULTIPARTITE COMMUNICATION

In [1] we introduced the multipartite entangled resource state as the string Fourier transform (SFT) $\mathfrak{F}_s$ acting on the ground state $|\vec{0}\rangle$,

$$|\mathrm{Max}\rangle = d^{\frac{1-n}{2}} \sum_{|\vec{k}|=0} |\vec{k}\rangle \ . \tag{1}$$

Here $\vec{k} = (k_1, \cdots, k_n)$, with $k_j \in Z_d$, and $|\vec{k}| = \sum_{j=1}^{n} k_j$. On the other hand, the $|\mathrm{GHZ}\rangle$ state was introduced in [14],

$$|\mathrm{GHZ}\rangle = d^{-\frac{1}{2}} \sum_{l=0}^{d-1} |k, k, \cdots, k\rangle \ , \tag{2}$$

and experimental work on $|\mathrm{GHZ}\rangle$ has been achieved [15, 16]. In fact $|\mathrm{GHZ}\rangle$ is the Fourier transform of $|\mathrm{Max}\rangle$, namely

$$|\mathrm{GHZ}\rangle = (F \otimes \cdots \otimes F)|\mathrm{Max}\rangle \ . \tag{3}$$

See §I and §III of [1] for relations between the Fourier transform $F$, the string Fourier transform $\mathfrak{F}_s$, and the entropy.

Let us describe our CT protocol in Fig. 1. Suppose a network has one leader and $n$ parties. Also assume that the $j^{\mathrm{th}}$ party can perform a controlled transformation

$$T_j = \sum_{l=0}^{d-1} |\ell\rangle\langle\ell| \otimes T_j(\ell), \tag{5}$$

where the control qudit belongs to the person $P_j$, and $T_j(\ell)$ can be an arbitrary multi-person, multi-qudit transformation on the targets. (The protocol of the controlled transformation $T_j$ is shown in Fig. 2.)

With these assumptions, the leader can perform a controlled transformation to all parties in the network,

$$T_c = \sum_{l=0}^{d-1} |\ell\rangle\langle\ell| \otimes T_n(\ell) \otimes \cdots \otimes T_1(\ell) \ . \tag{6}$$

This requires using a multipartite resource state $|\mathrm{Max}\rangle$ among the leader and the persons $P_j$. The leader has the common control qudit in $T$, and the $j^{\mathrm{th}}$ party performs the transformation $T_{j,l}$ for control qudit $\ell$. This protocol

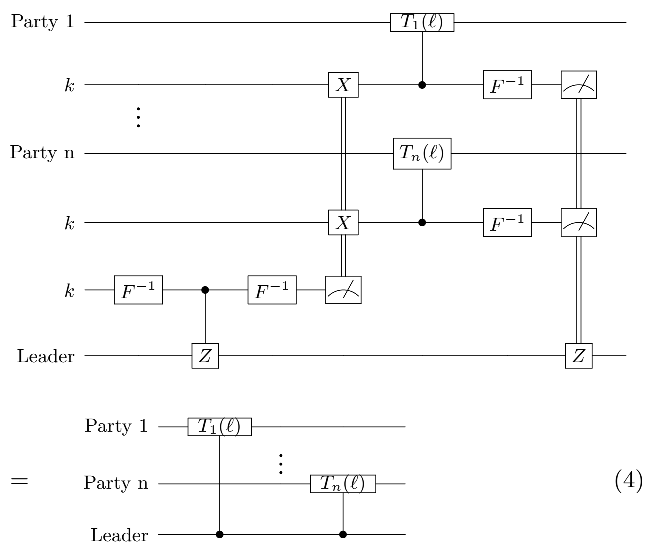


FIG. 1. CT protocol for controlled tranformations: The $k$'s arise from $|\mathrm{GHZ}\rangle$ in (2) for the leader and the persons $P_j$, $1 \le j \le n$. The output of the protocol is the multipartite, controlled transformation $T_c$.

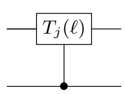


FIG. 2. Controlled transformations.

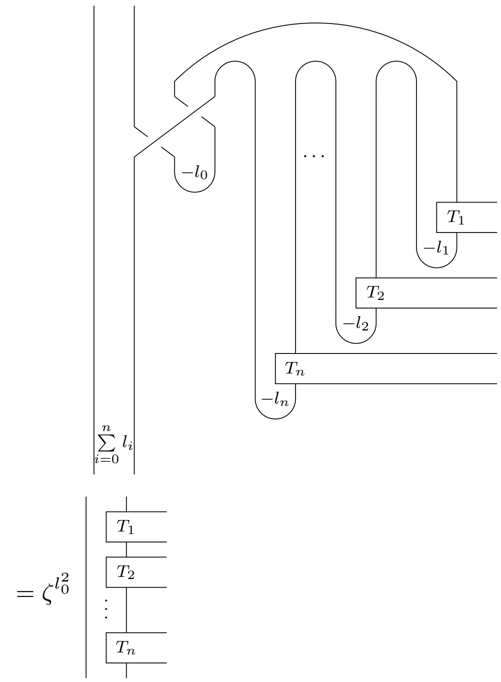


FIG. 3. Diagrammatic CT-protocol for $X$-compressed transformations.

costs one resource state $|\mathrm{Max}\rangle$ and $2n$ cdits. The time cost is the transmission of two cdits and the implementation of local transformations.

In [1] we analyze the BVK protocol [17] using holographic software. It would be interesting to analyze other protocols by this method, such as those in [10, 18–28].

## III. CT DETAILS

We say that a transformation $T$ is $Z$-compressed on the $i^{\mathrm{th}}$-qudit if $T$ is generated by Pauli $Z$ on the $i^{\mathrm{th}}$-qudit, and arbitrary transformations on the other qudits. Similarly we define $X$-compressed or $Y$-compressed transformations. Note that a transformation $T$ is $Z$-compressed on the first qudit if and only if it is a controlled transformation, namely $T = \sum_{\ell=0}^{d-1} |\ell\rangle\langle\ell| \otimes T(\ell)$.

We can switch between the three compressed transformations using $FXF^{-1} = Z$ and $GXG^{-1} = Y^{-1}$; see §II B of [1] for details.

We say that a transformation $T'$ is compressed on the $i^{\mathrm{th}}$-qudit if $T' = UTV$, where $T$ is $Z$-compressed on the $i^{\mathrm{th}}$-qudit and $U, V$ are local transformations on the $i^{\mathrm{th}}$-qudit. We give the CT diagrammatic protocol for $X$-compressed transformations in Fig. 3.

Using our dictionary of the holographic software, we give the CT algebraic protocol in Fig. 4.

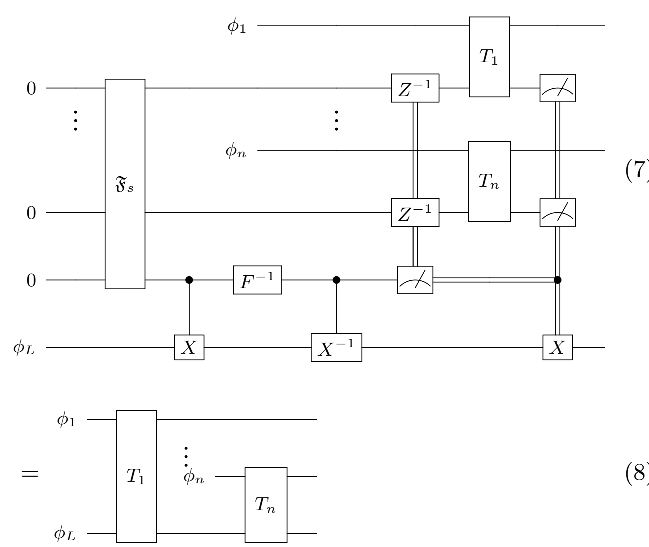


FIG. 4. CT protocol for $X$ compressed tranformations: The resource state $|\mathrm{Max}\rangle$ is expressed as $\mathfrak{F}_s|\vec{0}\rangle$. One can simplify the protocol by Fig. 5

Taking the conjugation of local transformations, we ob-

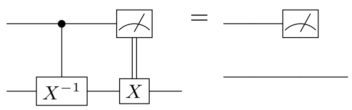


FIG. 5.

tain the CT protocol for compressed transformations. In particular, taking the conjugate of the Fourier transform $F$, we obtain the CT protocol for $Z$-compressed transformations (or multipartite, controlled transformations) in Fig. 1.

In the case with only two persons, the CT protocol says: Assume that a quantum network can perform a transformation $T$, which is compressed on a 1-qudit belonging to a network member Alice. Then Alice can teleport her 1-qudit transformation to Bob using one edit and two cdits. One can easily derive the swapping protocol, and the teleportation of the Tofolli gate from it.

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